Operation of Photomultiplier Tubes: Noise Considerations

1. Overview

A setup for light measurement is composed of a PMT with attached electronic circuitry. Both parts, PMT and electronic, generate noise. The noise in the anode current is caused by the statistical processes in the PMT and is not correlated to the electronic noise sources.
The total noise therefore is calculated:

$$V_{out,n} = (V_{PMT,n}^2 + V_{el,n}^2)^{1/2} \qquad (1)$$

with: $V_{PMT,n}$ = r.m.s. noise due to the PMT referred to output
$V_{el,n}$ = r.m.s. noise of the electronic part referred to output

Alternatively the noise contribution of the electronic part may be specified as input referred noise current $I_{in,n} = V_{el,n} / R_{conv}$ with R_{conv} = conversion gain resistor.

If one of the two terms $V_{PMT,n}$ and $V_{el,n}$ exceeds the other by more than a factor 2, the contribution of the smaller one to the total noise is less than 12%. Further reduction of the smaller term makes not much sense, because it is of negligable influence on the total noise.

2. PMT Noise

An excellent treatise on PMTs including all aspects of noise generation can be found in "BURLE Photomultiplier Handbook" (see section 5, /1/) and need not be repeated here, only the results are used.

The signal-to-noise ratio for the cathode current I_C (with dark current I_{dc} compensated) is given by:

$$SNR_{Cathode} = I_{C,signal} / I_{C,noise} = I_{pe} / [2q\ (I_{pe} + I_{dc})B_{eq}]^{1/2} \qquad (2)$$

with: $I_{pe} = \eta \times F_p \times q$ = average photoemission current
η = quantum efficiency of the photocathode
F_p = rate of photons impinging upon the photocathode
q = elementary charge
I_{dc} = average dark current
B_{eq} = equivalent noise bandwidth of the detection circuitry

It is assumed, that the PMT-gain μ is nearly noise-free (which is true for many PMTs if operated with high gain μ, see/1/ in sect.5). Therefore $SNR_{Cathode} \approx SNR_{Anode}$, eq. (2) gives the theoretical upper limit for the signal-to-noise ratio for integral current measurements with PMT´s.

The noise part in the anode current has a flat ("white") frequency spectrum and can be characterized by the noise current density:

$$i_{A,n} = \mu\ (2qI_C)^{1/2} \qquad (3)$$

with: μ = gain of the PMT
I_C = average cathode current

The anode current is converted to a voltage by means of a suitable network with conversion gain R_{conv} and the r.m.s. value of the output noise voltage due to PMT noise is given by:

$$V_{PMT,n} = R_{conv}\, I_{A,n} = R_{conv}\, i_{A,n}\, B_{eq}^{1/2} = \mu\, R_{conv}\, (2qI_C B_{eq})^{1/2} \qquad (4)$$

Note that the output signal is given by: $V_{out,s} = R_{conv}\, I_A = \mu\, R_{conv}\, I_C$

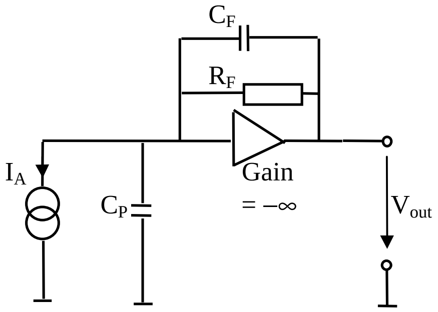
and hence: $V_{out,s} / V_{PMT,n} = (I_C / 2qB_{eq})^{1/2}$

Optimising the signal-to-noise ratio requires:

$I_C \uparrow$ (more light, less gain)
$B_{eq} \downarrow$ (bandwidth as small as possible with respect to the signal)

3. Electronic Noise

The anode current I_A is converted to a voltage by means of a resistor in the feedback path of an amplifier ("transimpedance amplifier"):



C_P = parasitic capacitance at the anode (PMT + amplifier + stray)

network transfer function:
$V_{out} = I_A R_F / (1 + j\omega C_F R_F)$
signal bandwidth: $f_{-3dB} = 1/ (2\pi C_F R_F)$

equivalent noise bandwidth:
$B_{eq} = 1/ (4R_F C_F) = (\pi /2)\, f_{-3dB}$ (5)

Noise sources in the electronic circuit are:

(a) Johnson noise of the resistor with noise voltage density $e_{R,n} = (4kTR_F)^{1/2}$ with T = absolute temperature. The resistor`s noise referred to output is $V_{R,n}$.

(b) Input referred noise voltage density of the amplifier $e_{in,n}$. This is a quality of the amplifier and it is assumed for simplicity, that the frequency spectrum of $e_{in,n}$ is white and that the amplifier exhibits negligable input current noise.
The amplifier`s noise referred to output is V_n .

The two noise sources (a) and (b) are not correlated, therefore they have to be added squared:

$$V_{el,n}^2 = V_{R,n}^2 + V_n^2 \qquad (6)$$

Quantitative expressions:

(a) The resistor noise is bandlimited due to C_F and transferred to the output with unity gain:

$$V_{R,n} = e_{R,n}\, B_{eq}^{1/2} = (4kTR_F B_{eq})^{1/2} = (kT/C_F)^{1/2} \qquad (7)$$

(b) The noise gain for the amplifier noise is given by:

$$H_n(j\omega) = (1+j\omega(C_F + C_P) R_F) / (1+j\omega C_F R_F) \qquad (8)$$

This transfer function is not bandlimited and approaches $(C_F + C_P) / C_F$ for high frequencies. In order to achieve a satisfying signal-to-noise ratio, an additional bandlimitation has to be implemented!

This necessary lowpass filter influences the signal as well, therefore it must exhibit a clean, overshoot-free step response to preserve the signal in the time domain. Due to the additional lowpass the overall bandwidth is reduced, expressions for f_{-3dB} and B_{eq} in eq. (5) do not apply for the complete current-to-voltage converter.

For low frequency applications with $f_{-3dB} < 1/(2\pi R_F(C_F+C_P))$ the increase in noise gain with frequency is of no effect, it can completely be suppressed by the lowpass.

For wideband applications, however, the noise gain increases proportional to frequency in the region $1/(2\pi R_F(C_F+C_P)) < f < 1/(2\pi R_F C_F)$. It is of greatest importance therefore to keep C_P as small as possible and to limit the bandwidth to the necessary minimum.

Because of these complex relations a closed expression for $V_{el,n}$ cannot be derived for the general case. WMT current-to-voltage converters are equipped with an internal overshoot-free filter the bandwidth of which can be set within certain limits according to the customer`s needs.
The r.m.s. value of the output noise, $V_{el,n}$, (or the input referred noise current $I_{in,n}$) is specified for certain standard configurations, for others the customer may extrapolate neighbouring data or contact the factory. Timing and noise specifications are valid with C_P = 10pF, a realistic value for a PMT including mounting and stray capacitance.
The equivalent noise bandwidth B_{eq} to calculate the transfer of the PMT noise - see eq. (2) - is typically $B_{eq} = 1.25\ f_{-3dB}$.

4. Calculations

For large cathode currents I_C the dark current I_{dc} is negligable and therefore $I_C = I_{pe}$. From (2) we obtain the expression for the signal-to-noise ratio:

$$SNR = I_C / (2qI_C B_{eq})^{1/2} = [I_C / (2.5qf_{-3dB})]^{1/2} = 50\,[(I_C / fA) / (f_{-3dB} / Hz)]^{1/2}$$

$$SNR_{dB} = \{10lg[(I_C / fA) / (f_{-3dB} / Hz)] + 34\}dB \qquad (9)$$

Eq.(9) gives the upper limit for the achievable signal-to-noise ratio, based exclusively on the statistics of photoemission in the cathode and assuming, that the PMT gain µ is noise-free and that the equivalent noise bandwidth of the voltage-to-current-converter is $B_{eq} = 1.25\ f_{-3dB}$.

In a log/log-plot eq.(9) defines parallel straight lines with slope +10dB per decade of I_C and 10dB spacing per decade of f_{-3dB} .

With $I_{C,max}$ given and $V_{out,max}$ intended for the output one obtains the overall conversion gain

$$V_{out,max} / I_{C,max} = \mu R_{conv}$$

Note: The PMT gain µ must be chosen as high as possible with respect to the properties of the PMT (in particular dark current increasing with voltage, i. e. with gain), because the PMT gain is nearly noise free and the electronic gain is not.

The output noise $V_{el,n}$ can be referred to the cathode current:

$$I_{el,C,n} = V_{el,n} / (\mu R_{conv}) \tag{10}$$

and the total cathode-referred noise current becomes:

$$I_{C,n} = (2qI_C B_{eq} + 2qI_{dc}B_{eq} + I_{el,C,n}^2)^{1/2}$$

$$= [(2.5qI_C f_{-3dB}) +(2.5qI_{dc} f_{-3dB} + I_{el,C,n}^2)]^{1/2} \tag{11}$$

For large I_C the first (..) term in eq.(11) predominates and the signal-to-noise ratio is given by (9) with 10dB slope per decade of I_C , for small cathode currents the first term is negligable and the SNR has 20dB slope per decade of I_C . The intersection of the two asymptotes occurs for cathode current I_{CS} where the two terms are equal:

$$(2.5qI_{CS} f_{-3dB}) = (2.5qI_{dc} f_{-3dB} + I_{el,C,n}^2)$$

$$I_{CS} = I_{dc} + I_{el,C,n}^2 / (2.5qf_{-3dB}) \tag{12}$$

Examples:

a) Low level, low frequency:

PMT: I_C = 2pA full scale; I_{dc} = 1fA
amplifier: V_{out} = 10 V full scale, $V_{el,n}$ = 0.1 mV; f_{-3dB} = 200 Hz

overall conversion gain: $\mu R_{conv} = 10\ \text{V}/2\ \text{pA} = 5 \times 10^{12}\ \Omega$
cathode referred electronic noise: $I_{el,C,n} = V_{el,n} / \mu R_{conv} = 2 \times 10^{-17}\text{A}$

From eq. (9) we obtain for large cathode currents: $SNR_{dB} = [10\lg(I_C /\text{fA}) + 11]$ dB

For small cathode currents, dark current I_{dc} and electronic noise $I_{el,C,n}$ play a role with the intersection point acc. to eq. (12):

$$I_{CS} = 1\ \text{fA} + [(4 \times 10^{-34})/(4 \times 10^{-19} \times 200)]\text{A} \approx 1\ \text{fA}$$

In this case, the electronic noise is negligable.

b) Medium level, medium frequency:

PMT: I_C = 100 pA full scale; I_{dc} = 1 fA
amplifier: V_{out} = 10 V full scale, $V_{el,n}$ = 0.4 mV; f_{-3dB} = 20 kHz

overall conversion gain: $\mu R_{conv} = 10\ \text{V}/100\ \text{pA} = 10^{11}\ \Omega$
cathode referred electronic noise: $I_{el,C,n} = V_{el,n} / \mu R_{conv} = 4\ \text{fA}$

From eq. (9) we obtain for large cathode currents: $SNR_{dB} = [10\lg(I_C /\text{fA}) - 9]$ dB

For small cathode currents, dark current I_{dc} and electronic noise $I_{el,C,n}$ play a role with the intersection point acc. to eq. (12):

$$I_{CS} = 1\ \text{fA} + [(1.6 \times 10^{-29})/(4 \times 10^{-19} \times 2 \times 10^4)]\ \text{A} = 3\ \text{fA}$$

In this case, the contributions of dark current and electronic noise are of comparable magnitude, but both are negligable:

The statistics of photoemission alone lead to $SNR_{dB} = 0$ for I_C = 8fA acc. to eq. (9).

c) High level, wide bandwidth:

PMT: $I_C = 100$ nA full scale, $I_{dc} = 5$ fA

amplifier: $V_{out} = 4$ V full scale, $V_{el,n} = 1$ mV, $f_{-3dB} = 3$ MHz

overall conversion gain: $\mu R_{conv} = 10\ \mathrm{V}/2\ \mathrm{pA} = 4 \times 10^{7}\ \Omega$

cathode referred electronic noise: $I_{el,C,n} = V_{el,n}/\mu R_{conv} = 25$ pA

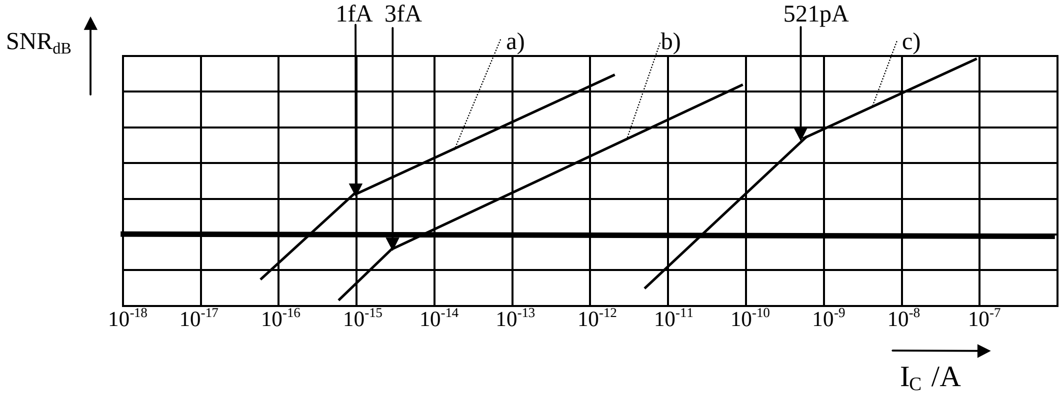
From eq. (9) we obtain for large cathode currents: $SNR_{dB} = [10\lg(I_C/\mathrm{fA}) - 30.8]$ dB

For small cathode currents, dark current I_{dc} and electronic noise $I_{el,C,n}$ play a role with the intersection point acc. to eq. (12):

$$I_{CS} = 5\ \mathrm{fA} + [(6.25 \times 10^{-22})/(4 \times 10^{-19} \times 3 \times 10^{6})]\mathrm{A} = 521\ \mathrm{pA}$$

In this case, the lower limit of detection is determined by the electronic noise.

Diagrams:



Shown are the asymptotes, the actual functions are -3dB below at the intersection point.

5. Literature

/1/ BURLE PHOTOMULTIPLIER HANDBOOK
Theory-Design-Application of PMTs
Available from BURLE, nowadays preferably on CD-ROM

/2/ BURLE ELECTRO-OPTICS HANDBOOK
Technical information on electro-optics for scientists and engineers.
Available from BURLE, nowadays preferably on CD-ROM

/3/ Henry W.Ott: Noise Reduction Techniques in Electronic Systems
John Wiley & Sons, 2. Ed. 1988
ISBN 0-471-85068-3

/4/ Glenn F.Knoll: Radiation Detection and Measurement
John Wiley & Sons, 2. Ed. 1989
ISBN 3-468-20943-4