PMT Noise Considerations

1. Overwiew

An excellent treatise on PMTs including all aspects of noise generation can be found in "BURLE Photomultiplier Handbook" , available from BURLE Industries Inc.

In the following only a brief discussion will be given of the special case, that the photons impinging upon the photocathode are Poisson-distributed, and that the integral anode current is to be measured. This is a standard task in practice.

Other cases, in particular questions of single or multiple particle output distribution or pulse-height resolution are not treated here, please refer to the mentioned literarure.

2. PMT Noise

Photon noise:
If the photons in a flux with rate F_p [s^{-1}] hitting the photocathode are Poisson-distributed, the average number of photons $\bar{n}_p$, arriving in a time intervall τ is equal to the variance σ_p^2 :

$$\bar{n}_p = F_p \times \tau = \sigma_p^2 \qquad (1)$$

The signal-to-noise ratio becomes:

$$SNR_p = \bar{n}_p / \sigma_p = (F_p \times \tau)^{1/2} \qquad (2)$$

Photoemission noise:
The physics of photoemission are somewhat simplified described by a single parameter η, the quantum efficiency. It can be shown, that for intervals $\tau >> 1 / F_p$ (i.e. for integral and not for pulse measurements) the signal-to-noise ratio is given by:

$$SNR_{pe} = (\eta \times \bar{n}_p)^{1/2} = (\eta \times F_p \times \tau)^{1/2} \qquad (3)$$

Dark current noise:
Dark current is generated by ohmic leakage (which usually can be reduced to a negligible value by proper selection of material, cleaning, and shielding) and thermionic emission of electrons, which follow a Poisson-distribution as well. While the dark current itself can be compensated (an appropriate input is implemented in SMT detectorheads), the fluctuation adds to the fluctuation of the signal:

$$\sigma_{th}^2 = \bar{n}_{th} \qquad (4)$$

At the cathode:

$$\sigma_C^2 = \sigma_{pe}^2 + \sigma_{th}^2 = \eta \times \sigma_p^2 + \sigma_{th}^2 = \eta \times \bar{n}_p + \bar{n}_{th} \qquad (5)$$

and:

$$SNR_C = (\eta \times \bar{n}_p) / (\eta \times \bar{n}_p + \bar{n}_{th})^{1/2} \qquad (6)$$

$$= \sqrt{(\eta \times \bar{n}_p)} \qquad \text{if} \quad \bar{n}_{th} \equiv 0$$

Werksstrasse 15
D-45527 Hattingen, Germany phone: (+49) 2324 506764 fax: (+49) 2324 506765 e-mail: wmt.elektronik@t-online.de

It should be emphasised at this point of discussion, that the signal-to-noise ratio in (6) is given by the properties of the PMT and the photon flux rate. For pulse measurements the dark spikes $\bar{n}_{th}$ may be partially or totally eliminated in some applications by means of coincidence detection or pulse height discrimination, for integral measurements (6) is the theoretical overall maximum for the signal-to-noise ratio and cannot be improved by any means of electronic signal processing.

<u>Noise introduced by variations of gain:</u>

Statistics of secondary emission:

A single electron striking the dynode D_i produces an average number of secondary electrons δ_i with variance σ_i^2 . By cascading k dynode stages the average number of electrons becomes:

$$\bar{m}_k = \prod_{i=1}^{k} \delta_i \qquad (7)$$

with variance:

$$\sigma_{mk}^2 = \bar{m}_k^2 \left[\sigma_1^2 / \delta_1^2 + \sigma_2^2 / (\delta_1 \delta_2^2) + \ldots. + \sigma_k^2 / (\delta_1 \delta_2 \ldots \delta_{k-1} \delta_k^2)\right] \qquad (8)$$

As to be seen from (8), the influence of the individual dynode stages onto the overall variance decreases with the distance from the cathode. If the first stage has high gain, (8) simplifies to:

$$\sigma_{mk}^2 \approx \bar{m}_k^2 \left[\sigma_1^2 / \delta_1^2\right] \qquad (9)$$

If in addition the secondary emission follows the Poisson-distribution, $\sigma_1^2 = \delta_1$ holds and the signal-to-noise ratio becomes:

$$SNR_k = \bar{m}_k / \sigma_{mk} \approx \sqrt{\delta_1} \qquad (10)$$

The noise contribution of the cascaded stages is very small in this case (first stage has high gain, secondary emission of first stage exhibits Poisson-statistics), the PMT gain is said to be noise-free, the signal-to-noise ratio is the same for anode and cathode of the PMT: $SNR_A \approx SNR_C$

For stages with equal gain ($\delta_i \equiv \delta$) and all described by Poisson-statistics eq. (8) becomes:

$$\sigma_{mk}^2 = \bar{m}_k^2[(1- \delta^{-k}) / (\delta -1)] \approx \bar{m}_k^2 /(\delta - 1) \qquad (11)$$

and

$$SNR_A \approx SNR_C \times \sqrt{(\delta -1) / \delta} \qquad (12)$$

Gain variation introduced by noise of the HV-supply:

A PMT with n equal-gain stages each with individual gain δ has the current gain $\mu = \delta^n$. For usual operating voltages δ is proportional to the applied dynode voltage V_D and this is a fraction of the applied HV. Therefore:

$$\mu = \text{const.} (HV)^n$$

and

$$\Delta\mu / \mu = n \times \Delta(HV)/(HV) \qquad (13)$$

Deviations of the HV from the ideal create an n-fold gain change.

Observation bandwidth

The noise calculations presented here refer to photons with average quantity $\overline{n}_p$ striking the photo-cathode in time interval τ. These calculations can directly be applied to pulse counting measurements. For current measurements it is more convenient to deal with currents instead of events and bandwidth instead of time intervals.

Since the impulse response of a PMT is a very short pulse, the frequency spectrum of the noise current can be considered as "white", i.e. independant of frequency within the frequency of interest and characterized by it´s spectral density i_n .

For a system with transfer function $H(j\omega)$ the r.m.s. noise current becomes:

$$I_{noise} = i_n \sqrt{\int_0^\infty |H(j\omega)|^2 \, df} = i_n \times B_{eq}^{1/2} \qquad (14)$$

The squareroot term is by definition the equivalent bandwidth B_{eq} .

A network implementing an observation window of length τ (e.g. a gated integrator) has an impulse response in form of a rectangular pulse of length τ and hence the network transfer function:

$$H(j\omega) = (1- e^{-j\omega}) / j\omega$$

with $B_{eq} = 1/2\tau$ calculated acc. to eq.(14). (15)

From eq. (3) the signal-to-noise ratio of the photoemitted electrons is:

$$SNR_{pe} = (\eta \times F_p \times \tau)^{1/2}$$

Substituting B_{eq} from (15) and the photoemitted current $I_{pe} = \eta \times F_p \times q$ with q = elementary charge one obtains:

$$SNR_{pe} = (I_{pe} / 2qB_{eq})^{1/2} = I_{pe} / (2qI_{pe}B_{eq})^{1/2} = I_{pe} / I_{noise} \qquad (16)$$

From (16) one obtains the familiar shot noise formula:

$$I_{noise} = (2qIB_{eq})^{1/2} \qquad (17)$$

Eq. (17) does hold for the signal current I_{pe} and for the dark current I_{dc} as well. So the total noise current density at the PMT cathode is:

$$i_{C,noise} = [2q (I_{pe} + I_{dc})]^{1/2} \qquad (18)$$

The signal-to-noise ratio for the cathode current I_C (with dark current I_{dc} compensated) is given by:

$$SNR_C = I_{C,signal} / I_{C,noise} = I_{pe} / [2q (I_{pe} + I_{dc})B_{eq}]^{1/2} \qquad (19)$$

with $I_{pe} = \eta \times F_p \times q$

Eq. (19) is the theoretical upper limit for the signal-to-noise ratio for integral current measurements with PMT´s.